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The effect of constraints due to fibres on the delamination of composites

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Stress criteria and energy criteria for crack propagation are examined. It is shown that attempts to improve the delamination resistance of composites are inevitably hampered by fibre constraints on matrix yield and flow. The toughness of the matrix in the absence of fibres is very roughly equal to twice the product of the matrix yield stress, the strain in the plastic zone, ε, and the thickness of the yield zone at the crack faces, t. For ductile matrices with a given yield stress, then, it is probable that toughness is very roughly proportional to t (i.e. ductile matrices are expected to have fairly uniformly high values of ε). However, when fibres are present (e.g. in a laminate) they severely restrict t, and hence the resistance to delamination. While t is less than the interfibre spacing, making the matrix tougher through increasing t directly affects composite toughness. Hence the resistance of the composite to delamination is directly proportional to matrix toughness. However, when t becomes so large as to equal the interfibre spacing, the development of matrix toughness is inhibited, and composite delamination resistance is little affected by further increases in matrix toughness. At this stage, additional increases in delamination resistance depend on increasing the matrix yield stress, rather than increasing the toughness of the matrix.

1. Introduction

Since it was realized that structures could be built using laminates made up from laminae having high volume fractions of aligned fibres, oriented in different directions, the benefits of these high-modulus, high-strength and low-density components have begun to be widely exploited. However, their brittleness, especially when carbon fibres are used, has been a problem [1]. This manifests itself in easy cracking across the fibres [2] and even easier cracking between the fibres. For example, with laminates the ease of delamination is a major problem [3]. It can be caused by relatively low energy impact, and results in loss of compressive strength. With opaque fibres, e.g. carbon, the damage, often being wholly inside the material, cannot easily be detected.

Because this is such a serious problem, the subject of delamination has been much studied. It is mainly a problem with the brittleness of the polymer used for the matrix, but a weak fibre–matrix interface is also a cause of low resistance to delamination. The advent of thermoplastics has helped to alleviate the problem, but the benefits have not been as great as had been hoped. It was expected that there should be a linear relationship between the work of delamination, and the work of fracture of the matrix. However, in practice this only occurred with relatively brittle polymer matrices, and when the tougher thermoplastic matrices were used, there appeared to be diminishing returns from increasing the matrix toughness, see Fig. 1 [4].

It is well known that for across-the-grain fracture (i.e. fracture which involves the breaking of a large proportion of the fibres) the matrix toughness is not a major component of the work of fracture of the composite [5]. This is because the fibres inhibit the plastic flow of the matrix. A similar effect must be operating in the case of delamination, and in fact evidence has recently been produced which suggests that delamination is controlled by matrix strength, rather than matrix toughness [6].

In this paper both the stress at the crack tip and the energy at the crack tip will be taken into account in order to determine the relative importance of each for propagation of delamination cracks. The crack opening mode will be considered.

2. Energy and stress criteria

First, examine materials which are more or less homogeneous. Griffith [7] first pointed out the importance of energy in the fracture process, and laid the foundation for the development of fracture mechanics by Irwin [8] and co-workers. This was based on the idea of the propagation of an infinitely thin crack, coupled with the idea, due to Orowan [9], that the important energy absorbing process was plastic flow in the material, close to the crack face, rather than simply surface energy. In the last 30 years or so as a result of this work, fracture mechanics has become a very useful tool for designers of large structures.

Because plastic flow occurs, there must be some rounding of the crack tip to relieve the high stresses there. Thus Wells [10] introduced the concept of crack opening displacement before crack propagation.

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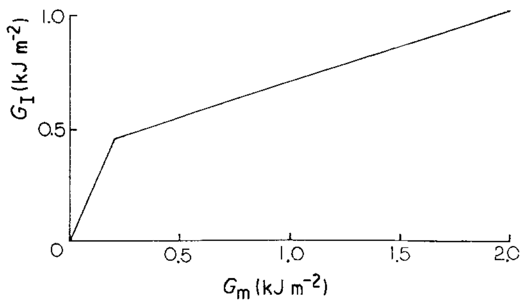

Figure 1 Work of fracture for delamination, G_I, plotted against matrix work of fracture, G_m, after Hunston *et al.* [4].

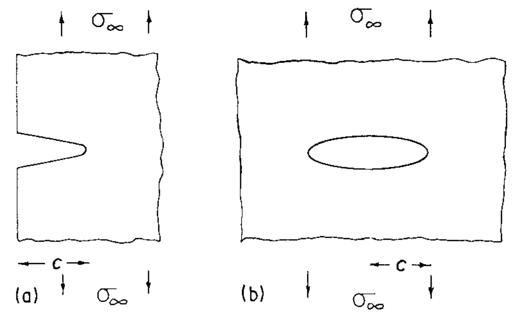

Figure 3 (a) Edge crack of length c, and (b) equivalent elliptical crack inside a piece of polymer.

Piggott [11] looked at the propagation of cracks of finite width, and showed that there exists a brittleness criterion. He showed that the fracture behaviour of metals and ceramics depends on the ratio of yield strength to Young's modulus. If this is greater than about 0.25% the material is brittle, if it is less than about 0.07% the material is ductile, and failed by tearing rather than fast fracture. Between these limits intermediate behaviour occurs.

The work of fracture can be directly correlated with the plastic work near the fracture surface. Thus Hall [12] showed, by successively etching and X-raying fracture surfaces, that there was a thin zone (thickness t) at each fracture surface, which, in the case of a fracture in an iron, was subjected to a yield strain, ε, which was about equal to 1.0 (see Fig. 2). If the yield stress of the steel is σ_y, we can write for the work of fracture, G, neglecting work hardening

$$G = 2\sigma_y t\varepsilon \qquad (1)$$

(remembering that the crack has two faces).

Probably the most important difference between a tough and intrinsically strong material like a structural steel, and a brittle but also intrinsically strong material like glass (the intrinsic strength of glass is evidenced by the strength of glass fibres) is the difference in the thickness of the worked zone, t. Hall and others have shown that it is moderately thick for steel; Marsh [13] showed that it had to be very thin for glass. Piggott [14] showed that inhomogeneity was an important factor; less inhomogeniety (as in the case of glass) tended to result in lower values of t.

In the development of the ideas of crack opening displacement and propagation of cracks of finite width, the yield stress played a role. This is clear from Equation 1, which relates the work of fracture to the yield stress. Thus an essential step in the development of a crack is the yielding of a small amount of material at the crack face. This concept of yielding can be applied to delamination of composites.

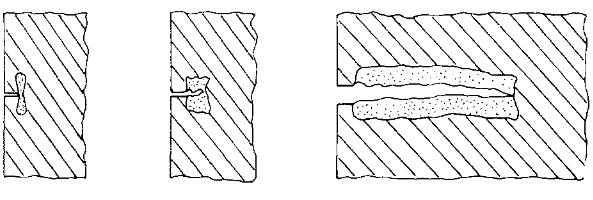

Figure 2 Three stages in the growth of a notch in a ductile polymer.

3. Crack tip stress field

Inglis [15] showed that in a homogeneous material there exists a stress field near the tip of an elliptic crack. It gives a maximum stress at the tip, σ_t, given by

$$\sigma_t = \sigma_\infty[1 + 2\,(c/r)^{1/2}] \qquad (2)$$

where σ_∞ is the stress applied to the material, c is half the crack length for an internal crack, and the full crack length for a surface crack (see Fig. 3) and r is the radius of the crack tip. Consider a crack with a sharp tip, such that $c/r \gg 1$ so that we can drop the 1 in Equation 2. In a laminate, where the crack is between laminae, the fibres will introduce an extra stress field into the matrix. Let the effect of this be to increase σ_t by a factor b. Now we can estimate the applied stress required to give yielding at the tip. This requires $\sigma_t = \sigma_{my}$ where σ_{my} is the matrix yield stress. Inserting these factors into Equation 2 and re-arranging gives, for the applied stress required to cause yielding

$$\sigma_{my}(r/c)^{1/2}/2b \qquad (3)$$

where $b > 1$. Now compare this with the Griffith–Irwin equation as written for anisotropic material.

$$\sigma_\infty = (E^* G_I/\pi c)^{1/2} \qquad (4)$$

where E^* is determined by the compliances of the composite [16]. For example

$$1/E^* = (S_{33}S_{11}/2)^{1/2}\,[(S_{11}/S_{33})^{1/2} + (2S_{13} + S_{55})/S_{33}]^{1/2} \qquad (5)$$

for a stress applied in the 3 direction, with a crack propagating normal to the 3 axis i.e. the crack opening mode. For a unidirectional laminate, E^* may be estimated using rule of mixtures expressions. For composites containing at least 50% stiff fibres (e.g. carbon, Kevlar or boron) $S_{11} \cong 1/V_f E_f$, $S_{33} \cong (1 - V_f)/E_m$, $S_{13} \cong -\nu_m/V_f E_f$ and $S_{55} \cong 2(1 - V_f)(1 + \nu_m)/E_m$ where V_f is the fibre volume fraction, E_f and E_m are fibre and matrix Young's moduli, and ν_m is the Poisson's ratio of the matrix which, for simplicity, has been assumed to be approximately the same as that of the fibres. For $V_f = 0.5$

$$E^* \cong 2(E_f E_m)^{1/2} \qquad (6)$$

Returning to Equations 3 and 4, it may be seen that they both have an inverse square root relation between σ_∞ and c. Thus if crack propagation proceeds by continuous yielding there is a pseudo toughness, G_{Ip},

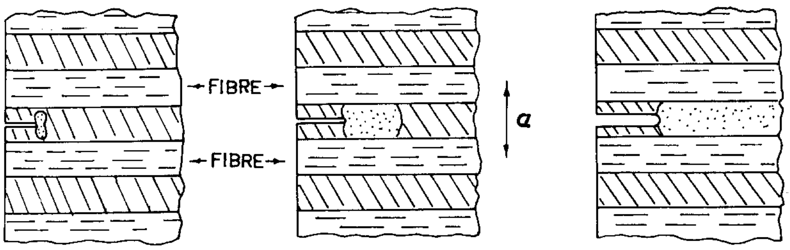

Figure 4 Three stages in the growth of a delamination crack in a composite.

obtained by equating the two expressions

$$G_{Ip} = \pi\sigma_{my}^2 r/4E^*b^2 \quad (7)$$

This pseudo toughness corresponds to the condition required for the stresses at the crack tip to be sufficient to overcome the resistance to yielding of the polymer at the crack tip.

4. Work at the crack tip

4.1. Work in the polymer matrix

We can use Equation 1 to estimate the work in the polymer at the crack tip, if t and ε can be evaluated. Fig. 4 illustrates how the fibres inhibit the extension of the plastic zone. It is unlikely that the zone extends beyond the axes of the fibres adjacent to the crack plane, i.e. $t \cong a/2$ in Fig. 4, where a is the fibre spacing. If the fibres are packed in a square array, and have diameter d, then

$$V_f = \pi d^2/4a^2 \quad (8)$$

so that $t \cong d(\pi/V_f)^{1/2}/4$ and Equation 1 becomes, writing σ_{my} for σ_y

$$G_{Im} \cong \sigma_{my}\varepsilon d(\pi/V_f)^{1/2}/2 \quad (9)$$

4.2. Fibre work

Delamination is facilitated if the fibres can debond easily. These fibre debonds can decrease the work in the matrix in proportion to the relative amount of fibre. (Hunston [4] reported some debonding with thermoplastic matrices.)

Also, the process envisaged in Fig. 4 involves some fibre flexure. The elastic energy in the flexed fibres must be supplied by the applied stress, and hence must contribute to the work of fracture. Suppose the fibres are flexed to a minimum radius R, as shown in Fig. 5. The region of flexure is expected to propagate along the fibres, keeping up with the rate of advance of the crack tip. The strain energy for unit distance of crack propagation is U_f, where

$$U_f = E_f I/2R^2 \quad (10)$$

where I is the moment of area of the fibres, i.e.

$$I = \pi d^4/64 \quad (11)$$

per fibre. Across unit width of crack front there will be $1/a$ fibres involved in flexure where a is obtained from Equation 8. It is likely that several layers of fibres will be involved. If the number of layers is n, then the total work (Equations 10 and 11) is

$$G_{If} = nE_f d^3(\pi/V_f)^{1/2}/128R^2 \quad (12)$$

Finally, let the fibre stress at the fibre surface due to flexure at radius R be σ_f. Because the corresponding strain is $d/2R = \sigma_f/E_f$ we can write

$$G_{If} = nd\sigma_f^2(\pi/V_f)^{1/2}/32E_f \quad (13)$$

5. Discussion

It is now appropriate to examine the relative contributions of the various processes discussed. To do this we need to make plausible assumptions about the various parameters used.

First examine the pseudo toughness arising from the need to overcome the resistance to yielding of the matrix, Equation 7. Let E^* be given by Equation 6. The maximum value of r and minimum value of b $(=1)$ will give the maximum value of G_{Ip}. Let r be equal to half the minimum spacing between fibres, i.e. $(a - d)/2$, which using Equation 8 comes to $r = d[(\pi/4V_f)^{1/2} - 1]/2$ and let $V_f = 0.5$, and $b = 1$. Then Equation 7 becomes

$$G_{Ip} \cong \pi\sigma_{my}^2 d[(\pi/2)^{1/2} - 1]/16\,(E_f E_m)^{1/2} \quad (14)$$

For carbon $d = 8\,\mu$m and $E_f \cong 233$ GPa (HTS fibres). For the polymer matrix σ_{my} is not likely to exceed 100 MPa, and 2.5 GPa is a typical value for E_m. This gives a maximum value of G_{Ip} of about $1\,\mathrm{J\,m^{-2}}$. Thus the crack tip stress field criterion is easily satisfied with a material having a work of fracture of $100\,\mathrm{J\,m^{-2}}$ or more.

For the work in the polymer matrix, G_{Im}, Equation 9, a value for ε is required. If a ductile polymer is used, it may be possible for ε to reach 1.0, as for metals. This gives a G_{Im} of about $1\,\mathrm{kJ\,m^{-2}}$ for $\sigma_{my} = 100$ MPa.

The work of fibre flexure, G_{If}, Equation 13 has its maximum value when the fibre flexure stress reaches the ultimate strength, i.e. about 3 GPa. Letting $n = 10$, this gives $G_{If} \cong 100\,\mathrm{J\,m^{-2}}$.

It is clear from these considerations that the work of matrix fracture is the most important component of

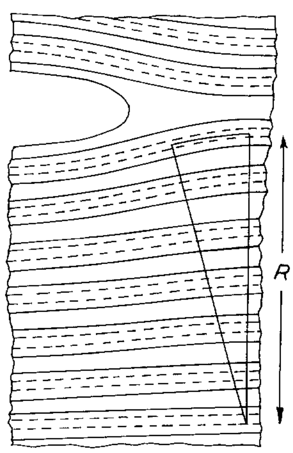

Figure 5 Fibre curvature close to a crack in a laminate.

the work of delamination. The work comes close to recent values for this (e.g. Gillespie *et al.* [17] gives 1.6 to 2.0 $kJ\,m^{-2}$), even though we are neglecting fibre pull-outs.

6. Conclusion

These considerations show that stress criteria and energy criteria for fracture are linked. In particular, plausible values for the work of delamination can be estimated from a matrix yielding process at the crack tip: Equation 9 gives results which are compatible with experimental values. The polymer yield stress assumed (i.e. about 100 MPa) and the strain at fracture of about 100% are close to practical values for these parameters.

As the plastic zone at the crack tip is limited by the adjacent fibres, the work of fracture of the matrix is an important factor, influencing delamination resistance only when the matrix fracture involves a thinner worked zone than the interfibre spacing. Thus for brittle polymers, with thin worked zones (the worked zone thickness may be estimated very roughly from Equation 1 if the plastic strain at fracture is known), the work of delamination can be greater than or equal to the work of fracture of the matrix. (Other contributors to the work of fracture are fibre breakage and pull-out. These depend on geometrical factors, and can only be estimated when the exact fibre geometry, e.g. straightness, is clearly defined.) With a tougher matrix, i.e. a thicker worked zone, toughness is limited because of constraints introduced by the presence of the fibres. Now, instead of the work of fracture of the matrix being one of the principal factors controlling the delamination, the matrix work during delamination is proportional to the yield stress and the maximum strain achievable in the matrix, in the fracture zone.

This leads to the conclusion that beyond a certain matrix toughness, further improvements in the resistance to delamination depend on producing polymers which have higher yield stresses, while still retaining very large strains to failure. This approach will only be successful if at the same time the fibre–matrix adhesion is strong enough to obviate the possibility of massive bond failure at the fibre surface.

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